Calculus Name_____

1. Given $x^2y - 2y + 5 = 0$
   a. Write an equation of the line tangent when $x = 1$.
   b. Find all points where the curve is horizontal, if any.
   c. Find all points where the curve is vertical, if any.

a) $y - 5 = 10(x - 1)$

b) $\frac{dy}{dx} = \frac{-2xy}{x^2 - 2} = 0$

$(0, \frac{5}{2})$

c) $\frac{dy}{dx}$ = undefined

$x^2 - 2 = 0$

$+2 \quad +2$

$x^2 = 2$

$x = \pm\sqrt{2}$

Plug into original to find y

$(\sqrt{2})^2 y - 2y + 5 = 0$

$2y - 2y + 5 = 0$

$5 = 0$

$(-\sqrt{2})^2 y - 2y + 5 = 0$

$2y - 2y + 5 = 0$

$5 = 0$

Since there are no y values there are no points with vertical tangents

2. Given $y^3 + 6 = xy$
   a. Write an equation of the line tangent when $x = 2$.
   b. Find all points where the curve is horizontal, if any.
   c. Find all points where the curve is vertical, if any.

a) Point: $(7, 2)$

$2^3 + 6 = 2x$

$8 + 6 = 2x$

$14 = 2x$

$7 = x$

Slope: $\frac{2}{5}$

$3y^2 \frac{dy}{dx} = x\frac{dy}{dx} + y$

$3y^2 \frac{dy}{dx} - x\frac{dy}{dx} = y$

$\frac{dy}{dx} = \frac{y}{3y^2 - x}$

$\frac{dy}{dx} = \frac{2}{3(2)^2 - 7} = \frac{2}{5}$

$y - 2 = \frac{2}{5}(x - 7)$

b) $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = \frac{y}{3y^2 - x} = 0$

$y^3 + 6 = xy$

$0^3 + 6 = x(0)$

$6 = 0$?

There are no points where there is a horizontal tangent.

c) $\frac{dy}{dx}$ = undefined

$\frac{dy}{dx} = \frac{y}{3y^2 - x} = 0$

$3y^2 - x = 0$

$3y^2 = x$

Tells the relationship at the point where there is a vertical tangent

Original

$y^3 + 6 = xy$

$y^3 + 6 = 3y^2 \cdot y$

$y^3 + 6 = 3y^3$

$6 = 2y^3$

$3 = y^3$

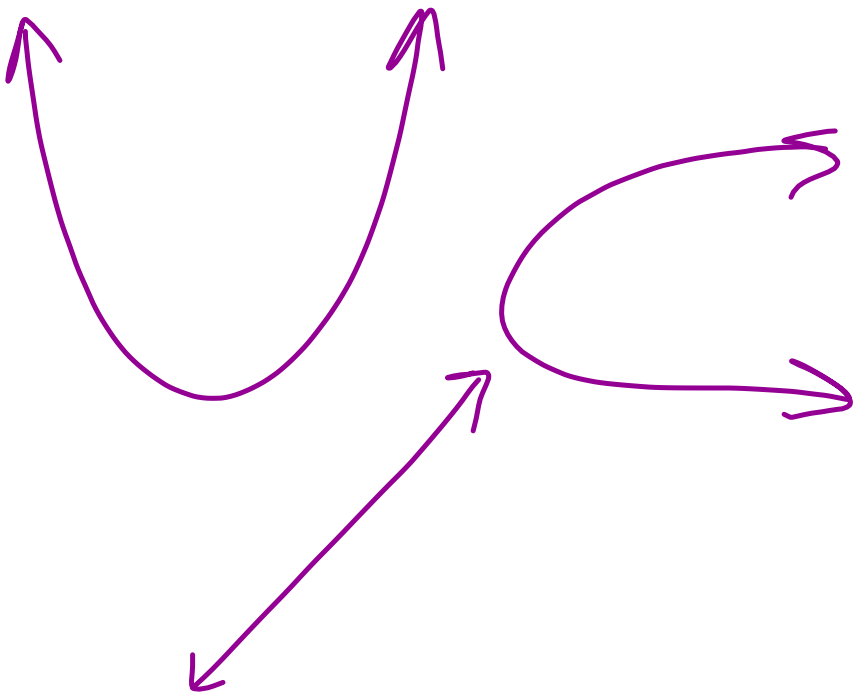
$\sqrt[3]{3} = y$

$3(\sqrt[3]{3})^2 = x$

$3^{\frac{3}{3}} \cdot 3^{\frac{2}{3}} = x$

$3^{\frac{5}{3}} = x$

$(\sqrt[3]{3^5}, \sqrt[3]{3})$



$x^2y - 2y + 5 = 0$

Point $(1, 5)$

$y - 2y + 5 = 0$

$-y + 5 = 0$

$5 = y$

$2xy + x^2\frac{dy}{dx} - 2\frac{dy}{dx} = 0$

$-2xy \qquad\qquad -2xy$

$\frac{dy}{dx}(x^2 - 2) = -2xy$

$\frac{dy}{dx} = \frac{-2xy}{x^2 - 2}$

use point $(1,5)$

Slope $= \frac{-2(1)(5)}{1^2 - 2} = \frac{-10}{-1} = 10$

Tangent @ $x=1$

$y - 5 = 10(x - 1)$

b) Horizontal tangents occur when slope $= 0$

* Numerator $= 0$

$\frac{dy}{dx} = \frac{-2xy}{x^2 - 2}$ $\qquad -2xy = 0$

$y = 0 \qquad x = 0$

Find Pts

$x = 0 \qquad y = \frac{5}{2}$

$(0, \frac{5}{2})$

$y = 0 \qquad x^2(0) - 2(0) + 5 = 0$

$0 - 0 + 5 = 0$

$5 = 0$

* There is not a point where $y = 0$

$0^2y - 2y + 5 = 0$

$-2y + 5 = 0$

$5 = 2y$

$\frac{5}{2} = y$